Connections between Chevallard's and Husserl's thoughts as drivers of a phenomenological pedagogies of mathematics

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Abstract: The article presents a theoretical study on the foundations of French didactics and phenomenology, searching for the approximations and withdrawals of Chevallard's and Husserl's proposals and possible unfoldings of the connections between their thoughts. Its content includes the explanation of what is being researched and the fundamentals of the method underlying the bibliographic investigation carried out inspired by phenomenological hermeneutics that permeates characteristics related to the intention of preparing the works, the views of science, the way of carrying out the theorisations, the conceptions of knowledge, of mathematics, and how human activity is understood by each of the authors. The articulation of the understandings that arise from the analysis shows the approximations and the distancing of their proposals, composing a backdrop for the gaze of the theoretical convergences related to mathematics and its construction, and considering the fact that the theorisations are based on the man-knowledge relationship and possible complementations on the understanding of what human activity is - and others that propel the elaboration of a phenomenological pedagogies of mathematics.

Keywords: anthropological theory of didactic; phenomenology; phenomenological hermeneutics; phenomenological pedagogies of mathematics

Introduction

> Obsessed, in fact, in a single culture, we are not only blind to the culture of others, but myopic when it comes to our own. The experience of alterity (and the development of this experience) commits us to see what we could not even have imagined, so our attention is struggling to fix on what is habitual, familiar, routine, and why we believe this is obvious. (Laplantine, 1987, p. 21, apud Ibarra, 2014, p.13)

It is very common in academic circles of mathematics education research that its members are grouped by elements of the area and develop their work following specific theorists, thus, playing the important role of theoretical deepening of paths of knowledge in agreement with the proper foundation, which carries with it the theoretical route chosen at each bifurcation that appears throughout their investigative journeys.

The theoretical essay presented here does not comply with the rule above, as it demarcates the confluence of intentions of researchers-participants and coordinators of different research groups, whose theoretical foundations are the phenomenology and the French didactic, who invested

efforts in developing a postdoctoral research conducted in the program of postgraduate studies in mathematics education at PUC-SP titled *Fundamentals of French didactic and phenomenology: Approximations and withdrawals between the proposals of Chevallard and Husserl and their unfoldings,* aiming to investigate the possibility of theoretical links based on the scientific and philosophical principles that gave origin to the positions and methodologies they adopted, that do not contradict each other and that, somehow, can leverage Chevallard's and Husserl's ideas and theories toward investigative territories that have not yet been explored. A rather naïve - if not daring – work, since apparently these authors have nothing in common. An audacious research seeking the roots of the ideas presented in their works, looking for confluences.

In spite of all the apparent mishaps, we were prepared to carry out the work, realising that the proposal to investigate possible approximations and estrangements from two elements of mathematics education is a major body of work and that the research presented here is a beginning of a possible dialectical movement full of meaning. Meaning, understood here as the guideline of an investigative journey still to be constructed, as it is open to receiving future contributions.

Investigative trajectory and its fundamentals

In line with Bicudo (1999) and Kluth (2013), the nature of this research is located in the region of inquiry of the Philosophy of Mathematics Education, as it puts the theoretical thoughts of the two authors in epoqué when questioning them in the perspective of their approximations and estrangements. We resort to a bibliographical investigation that bases its methodological fundamentals to understand and interpret texts on the philosophical hermeneutics of Gadamer (1997), Kluth (2005), and Kluth (2015).

Gadamer (1997), in weaving his philosophical hermeneutics, presents us a way of approaching human works when we intend to understand and interpret them, which is the interrogative mode intertwined with the possibility of the answer. In this procedure, rigor emerges from the "[...] discipline of questioning that allows for the construction of the *authentic, dialectical conversation* of the questions that arise in the course of research with their latent answers, comprehensible in human works¨. (Kluth, 2005, p. 41)

The analysis is therefore within the scope of the structure of the question and of the answer, because "if, on the one hand, the text must be understood as answering the question that asks, on the other hand, the latency of a response presupposes a question, the one the text answers". (Kluth, 2005, p. 41)

In this methodological approach, we understand a text insofar as we can formulate the question that the text seeks to answer, which is not necessarily the question that arises from and in the research. Hence, the search for adequate bibliography becomes fundamentally important and part of the research trajectory. The choice of authors' texts is intrinsically linked to the potential of the text to answer the questions that arise during the research.

Just as in Kluth (2015), the trajectory of the research presented here is composed of two moments. The first moment is intended to describe the fundamental ideas and their articulations present in the works of the authors in the form of texts, denominated here texts-solo of understandings.

The solo text of understandings of the first moment of the research presented here is composed of several hermeneutical studies on the work of the authors being studied. On the work of Chevallard, two texts were published: *A teoria antropológica do Didático: primórdios de uma trajetória direcionada à prática docente de ensino de matemática* and *Transposição didática em Chevallard: conceitos e teorização primordiais para a teoria antropológica do didático*.

To understand Husserl's works, texts elaborated and presented in Kluth (2015) were used, such as chapters of the doctoral thesis: *Estruturas da álgebra – Investigação fenomenológica sobre a construção do seu conhecimento/Structures of algebra - Phenomenological investigation on the construction of his knowledge* and unpublished works on abstraction and thinking in Husserl.

The second moment consists of the hermeneutic reading of the solo texts aiming to understand their meanings and to formulate questions whose answers about the approximations and estrangements from the authors' foundations are implicit in them.

From the hermeneutic reading of the solo texts mentioned above, we ask the following questions the solo texts answer about the approximations and estrangements from Chevallard's and Husserl's fundamentals: What was the authors' intention when elaborating their work? What view of science underlies the theory elaborated? How is the theorisation conducted? What is the authors' conception of knowledge? How do the authors conceive and interpret mathematics? Under which aspects do they understand human activity?

As the answers to these questions are intertwined, not to lose the whole that they reveal, we have chosen to bring them in the form of descriptive text containing the answers and analytical transition syntheses that subsidise the view of the theoretical links of the thinking of both authors.

Next, we will point out the approximations and the estrangements of ideas viewed in the analysis of the works of the authors and their unfoldings.

Approximations and estrangements between Chevallard's and Husserl's theoretical proposals and their unfoldings

Yves Chevallard, born on May 1, 1946 in Marseilles, France, writer, mathematician and researcher of didactic transposition in the field of didactics, introduces us to the development of a theoretical work that turns to scientific knowledge - *wise knowledge* - when merged with the intention of teaching it in school settings, becoming the *knowledge to be taught* and, consequently, *taught knowledge*. One of his works is the anthropological theory of the didactics, which places mathematical activity in the set of

human activities and social institutions. It has as its project the modelling of "human practices": mathematical practices or human actions of didactic nature.

Edmund Gustav Albert Husserl was born on April 8, 1859, Prosttejov, Czech Republic, and died on April 27, 1938, Freiburg im Breisgau, Germany. He was a mathematician and a philosopher. Husserl established a phenomenology school, which is dedicated to reflecting on the positivist orientation of science, introducing a new branch of philosophy, going through a critique of historicism and psychologism in the context of logic and the theory of knowledge.

The biographical fragments presented above make us wonder how far the ideas of these two entrepreneurs were in creating new ways of reflecting on knowledge, considering the reality that surrounded them and the chronological distance that separates their existences. But these assumptions may be exhausted when we seek the roots of their thinking, directing them toward the fulfilment of the purposes of mathematics education.

We begin by explaining the understandings arising from the analysis carried out in the research on the approximations and estrangements of Chevallard's and Husserl's ideas, focusing their intentions as researchers presenting answers to the question: What was the authors' intention when they created their work?

While Chevallard (1982;1992;1996;2002) focuses on the established scientific knowledge - in the sense of being institutionalised - in its relation with man, a fact that subsidises all the argumentation that justifies the anthropological characteristics of his theorization from the theory of the didactic transposition that culminates in the anthropological theory of the didactic, Husserl (1980; 2006; 2012; 1996) focuses on the description of the knowledge-building movement that results in the institutionalised knowledge and how the encounter of man with this knowledge in its construction movement occurs.

That is, Husserl's journey seeks the ontological of the knowledge, preserving coherence with his epistemological questions, and of how the episteme is becoming a cultural-scientific product. On the other hand, Chevallard's journey reveals, in the analysis of the research presented, to focus the institutionalised knowledge on "how" it is treated in the course of creating a didactic that presents it, taking organisational elements that compose its epistemology (technique, technology and theories) to weave an analysis that guarantees its preservation, presented in the anthropological theory of the didactics.

Both theorisations are based on the relation man-knowledge and there is a common concern in them of preservation of the established mathematical knowledge mediated by elements of the epistemology that cross the logic. In Chevallard (1982), preservation refers to what he called techno-cultural *(technoculturel),* which can be lost in the process of the didactics of mathematics, while in Husserl (2012), preservation concerns the sense of the world transmitted and not explained in scientific traditions.

Hence, we affirm that the focus of interest of the authors, when dealing with questions related to knowledge, has apparently divergent directions: Chevallard wants to preserve scientific knowledge, consistent with aspects of his epistemology, while Husserl wants to preserve the sense of the world not explained in it. However, we cannot fail to observe that both authors show a deep interest in conserving origins: Chevallard has as its starting point the established knowledge and its organization for didactic purposes, and Husserl seeks the constitution of the established knowledge, aiming at the construction of a formal *ontological* knowledge theory.

Our studies on Chevallard's thoughts, in his most diverse works, lead us to assume that those responsible for the preservation should be the agents of the educational system as creators and producers of the didactic choices. In Husserl, the agents of the preservation of the sense of the world should be, in our view, the scientists themselves when reflecting on their investigative methods and the resulting products.

In the works of the both authors Chevallard (1982) e Husser (1980), there is a latent and announced desire to depart from an analysis of the transmission or construction of knowledge that is strongly focused on psychology, as developed while writing their works.

What has hitherto been exposed about the authors' intentions in elaborating their theories suggests that there is a possibility that they complement each other, since Chevallard adheres to the trajectory of scientific knowledge (*wise knowledge*) to be delineated in *the noosphere in knowing to be taught* and transmitted within the system of teaching as *taught knowledge*, whereas Husserl concentrates on the constitution of knowledge, with emphasis on the ontological aspects of its constitution, its unfolding in epistemic aspects, and its permanence in the traditions. From this finding, we ask: Will this suggestion of intentional complementation continue in the same way their theories were elaborated? Are there confluences between the roots that sustain them and their ways of theorising and making science?

According to our analysis, Chevallard (1982) shows a deep interest in following the implementation of his ideas in the dynamics of the educational system, seeking information in its agents and incorporating the results obtained in his analysis, which gives his theorisation of the didactics of mathematics, didactic transposition and anthropological theory of the didactic an experimental bias characteristic. Husserl (2006), on the other hand, weaves a philosophical thinking that culminates in phenomenology, bearer of worldview, man and knowledge, resulting from an intentional analysis called *phenomenological reduction*. And, while he has provided answers to many criticisms received, these are always theoretical answers to theoretically formulated questions.

Underlying the way the authors elaborate on their theories, are concepts about what science is and its specific ways of constructions. Let us visit, even briefly, the way of theorising and the ideological supports of both authors, thus exposing responses contained in the solo texts to the questions: What view of science underlies the theory elaborated? and, Which is form of theorizing it?

According to Chevallard (1982, 1991b), to construct a scientific theorisation, a very well determined object of study is needed. In his theorisation about didactics of mathematics, the author defines it as the *techno-cultural* present in scientific knowledge, for example: that of mathematical knowledge, whose presentation in the mathematical activity in and from the classroom should be the most genuine.

Theorising the object of study of a science in the author's words:

¨The first theorisation work is marked by the reuse, ordered for a specific purpose, of existing materials. It is in this sense a bricolage activity (in the s Levi-starsien sense of the word), whose essential emphasis is the metaphor. For it is by the metaphorisation of an a priori heterogeneous set of old existences that the new emerges - that is, the theoretical construction itself, in its autonomy, producer of knowledge and generator of meaning¨. (Chevallard, 1991b, p. S/d)

In this passage, we understand that the author's conception of science and his way of doing science are explicit, centred in the aspect that describes the compilation of existing materials around a purpose, whose method used for such task is the metaphorisation, that gives space to the new. In this way, the author explains the didactics of mathematics as a science in an anthropological approach, which is attached to aspects borrowed from general anthropology.

¨The author in question takes as a starting point the Anthropology of Mathematics as a specialised field of Anthropology, in the sense that it is the study of man (and society) in contact with Mathematics and, particularly, the study of man producing Mathematics¨. (Kluth; Ag Almouloud, 2018, p. 11-12)

In other words, Chevallard (1992, 1996) dedicated himself to the study of man in contact with already established mathematics, however, he did not deal with the study of man producing mathematics, that is, he did not deal with the constitution of this knowledge process. He gave up this anthropological aspect, because "it is an issue that hitherto constituted, in an arbitrarily limiting way, the insurmountable horizon of the epistemology of mathematics, known to us all." (Chevallard, 1992, p.89). By justifying it, Chevallard places the didactics of mathematics as a subdomain of anthropology of mathematics, which studies man teaching and learning mathematics.

The core of the above thinkings will be shaped in the theorising of the didactic transposition, founded on the idea that to every contextualisation corresponds a decontextualisation. Chevallard (1982) radicalises this position, when he states: "All knowledge taken in *statu nascendi* is connected to its producer and makes, so to speak, part of it. Their sharing in the academic community presupposes a certain degree of depersonalisation, which, by itself, allows the disclosure of knowledge." (p.7)

For the author, the scientific knowledge assumes different applicabilities and, consequently, new meanings are attributed to it. Therefore, in each of these contextualisations different functions will be fulfilled. Within the didactic framework, it will perform functions that include the reproduction

and representation of knowledge, without being subject to the same restrictions of other social functions. It is in this set of constraints that, in the didactic transposition, the projections of *wise knowledge* - scientific knowledge, in which it eventually loses the intrinsic coherence with its origin- are constituted.

¨The knowledge produced by the didactic transposition will be a knowledge exiled from its origins and cut off from its historical production in the sphere of wise knowledge; legitimising itself as a taught knowledge, as something that was not from any time, from any place, and which does not legitimise itself by the appeal to the authority of a producer of any kind¨. (Chevallard, 1982, p. 4)

To theorise didactic transposition, Chevallard (1991b) claim that, although theory is a construction, an artefact aimed at enabling the understanding and study of the real, he would not discuss the real, because, for this, he would have to be restricted to a discussion about the abandonment of the representationist illusion.

This means that the real fact, in itself, is not scientist's centre of interest and that the didactics of mathematics seeks to understand and explain on scientific bases the "anthropological systems", that is, systems that involve, intrinsically, humans engaged in didactic life, which justifies Chevallard's (2013) thinking the relationships of facts versus phenomena, considering that

"/.../ a phenomenon cannot be defined on an empirical basis. Phenomena are *theoretical constructs.* To put it simply, I will say that the realm of phenomena is the theoretical counterpart to the multifaceted world of empirical facts". (Chevallard, 2013, p. 5).

We do not intend here to describe step by step the theoretical trajectory of the author that culminates in the theory of didactic transposition and the anthropological theory of the didactic, but to alert to the fundamentals that are being articulated for the construction of a theoretical model based on the relationship of the subject (individual or person) with an object, understood in the development of his work as: "every intentional product of human activity is an object¨ (Chevallard, 2002, p. 1), and in the relationship of the institution (institutions in general, schools, classroom etc.) with the object, always considering that "An object exists only because it is an object of knowledge". (Chevallard, 2002, p. 128).

At the beginning of the construction of the theoretical model that explains the "anthropological system" by the relationship between individual and object, the author states: "I would like to point out, in passing, that with regard to the notion of object, I know of few elaborations in which the theorisation I present here can be inspired, except for, perhaps, the Husserlian phenomenology and some of its offspring". (Chevallard, 1996, p. 129).

Although Chevallard, in the quote above, declares some confluence of his notion of object with that put in the Husserlian phenomenology, he does not provide details or indications about what constitutes this confluence, nor does he explain the constitution of this object for the individual; in this way, we understand that the notion talks about the object put in the noesis-

noema relationship. An intentional object, fruit of the relationship of man with the world, entitled noema, constituted and constituent of consciousness, that takes place in phenomenology by the *phenomenological reduction*, which is the analytical procedure of science of the essences presented by Husserl (2006).

Opposing the "natural" sphere of knowledge, Husserl (2006, 2012) says that empirical sciences are sciences of "facts". In his analysis, he asserts:

¨Although certain natural laws may be valid, by virtue of which, if such and such actual circumstances are factual, such and such certain consequences also have to be, these laws nevertheless express only factual regulations which could have an entirely different content, and already presuppose, as beforehand inherent in the essence of the objects of possible experience, that, taken in themselves, these objects regulated by them are contingent.¨ (Husserl, 2006, p. 35)

Following this line of critical thinking about how the empirical sciences are constructed, Husserl states that essence (eidos) is a new species of object given in the intuition of essence as pure essence. For this author, the realisation that the generalisation of the concepts of intuition and object is correlative and interdependent is not a finding, but a requirement of the nature of the things. In this perspective, he considers that:

¨Empirical intuition, and, in particular, experience, is the conscience of an specific object and, as an intuitive consciousness, "it brings the object to donation": as perception, it brings it to the original donation, to the consciousness that apprehends "originally the object in its "flesh and blood" ipseity. Exactly in the same way, the intuition of essence is consciousness of something, of an object, of something for which the gaze is directed, and which is "given" as being itself¨. (Husserl, 2018, p. 37)

In this way, Husserl speaks of two kinds of intuition: the empirical intuition and the intuition of essence that are realised in the perception. A very illustrative example of these species of intuition is given by Husserl (2012) in the work Investigações lógicas/Logical Investigations. We reproduce it here. Suppose we are in front of a coloured vase, for example, a red vase. We see the red of the vase, the individual red, but we also see, in the red of the vase, the redness of this red, which covers the whole category of the red colour. The intuition of redness puts us in the presence of all reds, shows us the essence of red, which occurs only in the presence of at least one red. This is a very important point of Husserl's work (2012), where we find that the abstraction, which leads us to the categories of species, is also a moment of perception, a moment in the noesis-noema relationship, which is based on the man-world relationship, where world is understood as the natural world, the instituted established world and the world in constitution. The relationship is, therefore, a moment of encounter of the man with objects liable to be known, therefore triggers of acts of conscience aiming at something in order to know it.

In order to understand better Husserl's (2006) statements, which present a science of essence, which has the essence as its object, it is necessary to clarify, even briefly, the procedure that underlies his way of searching and

thinking the world that gives support to his theory titled *Phenomenological Reduction*.

Phenomenological reduction is an analytical procedure that seeks to look at the object as if it were the very first contact with it, aiming at recovering the fruit of the intuition generated in the relationship with it. Thus, in the first moment of the reduction, a disconnection (Abschaltung) of the knowledge already established about the object is made, so that the object is shown as *what* it is, that is, in its essence. According to Husserl (2006), each of those *what* can be "put into idea", hence the use of the word *eidos* as synonymous with essence in Husserl's work.

From the *phenomenological reduction* of the empirical sciences carried out by Husserl (2006), we can state: "The empirical or individual intuition can be converted into a *view of essence (ideation)* - a possibility that should not be understood as an empirical possibility either, but as a possibility of essence" (p.35).

In our understanding, the *phenomenological reduction* allows the gaze of a possible transition between the empirical or individual intuition and the intuition of essence, even considering that both species of intuition are distinct by principle, since, in the species of the essential intuition, there is an important portion of individual intuition, and it can always turn to something individual and, on the other hand, the individual intuition, in effecting the ideation, will be able to direct its view to the corresponding essence. Therefore, the nexus that occurs between the individual object and essence is itself eidetic.

We understand that finding the possibility of transit between those first materials of knowledge, described as intuition, the possibility of transit between empirical science and science of essence will also be true. Husserl (2006) explains this by highlighting his concept of eidetic science, analysing ways of building science.

The author states that the natural scientist observes, experiences, and ascertains existence in accordance with experience. Experimenting is a founding act for him, never replaceable by mere imagination. Yet the scientist - constructor of the science of essences, considered pure (rein): as pure logic, as pure mathematics, as the pure doctrine of time and others- has all the steps of thinking, independent of any facts. Even when the geometer draws his figures on the blackboard, he investigates "idea possibilities" and states-of-essence, it is not the experience, but the intuitive apprehension of the essence, the ultimate founding act. "[...] It is so in all eidetic sciences. " (Husserl, 2006, p.43-44)

Husserl (2006) discusses the possibility of transit between the scientific mode of experimental science and eidetic science, stating that "the *sense* of eidetic science excludes, by principle, any and every incorporation of the cognitive results of the empirical sciences" (p.44); he justifies his assertion by stating that from facts always only facts will follow. He further adds that all eidetic science is, by principle, independent of the science of facts and vice versa. However, there is a relationship between them. For the author, no science of facts can develop as a science, regardless of the eidetic sciences, whether formal or informal. "For, in the first place, it is manifest

that an empirical science, wherever it makes mediated judgement foundations, must proceed according to formal principles treated by formal logic" (p. 44), that is, it is subject to the essential laws of the *general objectivity*, which connects it with all formal-*ontological* disciplines. "And secondly, it should be added that every fact includes a material eidetic substrate, and all the eidetic truth inherent in the pure essences contained therein must result in a law to which the given phatic singularity is subjected, as well as every possible singularity generally." (Husserl, 2006, p. 44).

We understand that what is being affirmed by the author is that it is implicit in the science of facts or science of experience a "residue" that will enable the transit between that science and the science of essence. "It can also be expressed thus: every science of facts (science of experience) has essential theoretical foundations in eidetic ontologies" (Husserl, 2006, p.44).

Husserl (2012) in *A crise das ciências europeias e a fenomenologia transcendental*/*The crisis of the European sciences and transcendental phenomenology* deals with the roots of this transit by elucidating the concept of world-life (Lebenswelt) and how the *sense of the world* is transmitted in the traditions, proposing a retroactive way of questioning the knowledge established by ideation, therefore apodictic, revitalising the sense of world transmitted there.

We understand that what has been stated so far is the basis for the following analysis.

Like Chevallard, Husserl also admitted the inseparability of fact and phenomenon, but the tendons of this link are explained by different perspectives in their work, generating different purposes and modes of theorising. Chevallard, at the very first moment of the construction of his theory, reuses the assertions and categorisations and groupings scientifically established or institutionalised, articulating them in a metaphorical way for the purpose of theorisation of the object of the science he constructed, the didactics of mathematics, in an anthropological approach. He adds to these articulations analyses of empirical data from experiences of agents of the educational system. In this way, Chevallard inserts his work in experimental science.

Husserl proposes that the *phenomenological reduction* be made in search of the essence of the object in question. He proposes, at the first moment of his theorising, the abstention of any knowledge established on the object analysed, as be carried away by the content of intuitions, elaborating an intentional analysis, aiming at constructing his own results that, later, could be interspersed by possible results of ideation put into the instituted knowledge.

Undoubtedly, these are ways of theorising that go hand in hand. From them, concepts that do not dialogue emerge, such as the concept of phenomenon that, for Chevallard, are theoretical constructs and that, for Husserl, is what is shown in the noesis-noema relationship, and that is not exhausted in the intentional analysis proposed by *phenomenological reduction*.

Figuratively, the phenomenon is, for Chevallard, something that can be imprisoned in theory, as a theoretical construction, since the author seeks a definition or theorisation that explains it, whereas for Husserl, the phenomenon is something that does not allow itself to be imprisoned in the *phenomenological reduction*, although it is subject to comprehension and this is subject to complementation. That is why the intentional phenomenological analysis is always a descriptive and never a conclusive analysis.

Depending on what is proposed as an object to be investigated scientifically, whether essence or facts, and if we consider the Husserlian analysis on the eidetic sciences and their interferences in the construction of the empirical sciences, the conceptual estrangements between the proposals analysed do not hinder the transit among them, they only show to be doubtful anchorages to become support for theoretical links. Thus, we understand that there is a possibility of a link between Chevallard's proposal on the didactics of mathematics as an experimental science, which passes through ideation, and Husserl's proposal in relation to the construction of a science of essence. This assertion is supported, for the moment, by the fact that it is Chevallard's theorisation - inspired in part by the noesis-noema relationship- that is being considered here as an ideation, and by the fact that what is expressed from it in Chevallard's texts considered for this research would not inhibit the search of essence of the encounter of man with the mathematics established in teaching and learning situations.

Although the confluence between experimental science has been views - didactics of mathematics and eidetic science with respect to the anthropological system, focusing on the encounter of man with knowledge as an ideation, which can be examined from Husserl's standpoint - some gaps of this confluence remain, since Chevallard, although taking the subject-object relationship as possible, and although it is the generator of his theoretical model in the theory of didactic transposition and the anthropological theory of the didactic, does not clarify "how" it occurs as a moment of knowledge. From this, we remain with the questions: How do the authors refer to knowledge and its construction? And particularly about the mathematics body of knowledge? Would they be compatible? Que nos remetem

What is the authors' conception of knowledge? How is mathematics conceived and interpreted by the authors?

For both authors, the knowledge of the same object can be constructed from various perspectives, remembering that, for Chevallard, object is only any product of human activity, and, in Husserl, beyond this, the object itself can also be considered a phenomenon.

For example, for Husserl, the number is a product of human activity that can be seen, itself, as a phenomenon, when asked about its origin, when we insist on describing its ontology in the human-world relationship and we reflect on its construction trajectory as mathematical ideality, while for Chevallard, on the contrary, the object is never the phenomenon, but the theoretical constructions that hover over the object. In this way, the number is taken only as a product of human activity, and the phenomenon is the theoretical constructs that explain it.

In the development of his theories, Chevallard makes use of the word knowledge for a quite generic designation, which includes to know how-to-do without the scientific knowledge. Nevertheless, to refer to knowledge as something elaborated with apodictic characteristics, the author uses the term *wise knowledge*. In this way, scientific knowledge is inserted in the spectrum of this knowledge and its objects are explained by the author as:

¨[...] objects that can be *apprehended*, and can be *taught*; more, that cannot be *known* without having been *learned*. On the other hand, they can be *used* and to exist, they must be *produced*. [...] Any S knowledge is associated with an institution of production of S […].¨ (Chevallard, 1992, p.107)

Chevallard, as already explained in this article, by exposing his understanding of the knowledge established as *wise knowledge* in the process of didactic transposition, emphasises that this undergoes interferences coming from the contextualisation of the knowledge in the educational system, that entrusts it new functions it must fulfil that extrapolate or differ from the functions this knowledge exercises on the body of knowledge where it was generated, at the risk of losing its historical and epistemological origins.

Husserl (2012), in the *Crise da ciência europeia/Crisis of European Science*, also detects a "wearing" movement of the roots of knowledge in the very knowledge constructed by the methodological procedures of the modern sciences, in elaborating the phenomenological reduction of the knowledge constitution, described as a movement that begins with the relationship between man and the world as a noesis-noeme relationship, therefore, a knowledge rooted in the intentional man-world relationship and how the product generated by this relationship is transmitted from generation to generation, as a tradition, revealing its historicity. There is, therefore, in this phenomenological description, an *Apriori* structure, given as *original evidence*, that is, the conscious apprehension of an entity in its original being-there, driver of eidetic achievements that transmit the sense of the world learned when in the human- world relationship.

But Husserl (1997) warns: "The original evidence cannot be confused with the evidence of axioms; because the axioms are, in principle, results of an original formation of meaning and already have original evidence behind them" (p. 450).

In other words, axioms are expressions of the idealisation of the ideal objectivity that occurs throughout the historicity of the construction/constitution of the ideal object taken in phenomenology as a *synthesis of transition*, bearer of essence as a sense of the world. In this sense, each *transitional synthesis* constitutes the universal *Apriori* of Husserl's history (1997), since it transmits the historicity of ideality and exposes the guiding thread of its construction in the phenomenological reduction. We can say that Husserl, throughout his work, weaves elements that can compose a philosophy of mathematics, which gives answers to ontological, epistemological and applicability questions of mathematical knowledge.

Thus, because both authors establish mathematics from its own body of knowledge in a non-contradictory way, since they assume their mode of development, their epistemology, the formal logic intrinsic in it and its language in the different stages of its evolution, we can affirm that there is a certain confluence and coherence of reasoning regarding mathematics.

However, the authors privilege aspects of mathematical knowledge that are being asked in the course of theorisation to meet the purposes of their work or, still, submit the same aspects to different analyses. Let us do an exercise to exemplify this finding, considering some of Chevallard's citations when discussing the function of *technology* in the accomplishment of didactic tasks from the perspective of the anthropological theory of the didactic.

Considering the statement: If 8 caramels cost 10 francs, 24 caramels, i.e., 3 times 8 caramels will cost 3 times more, i.e., 3 times 10 francs. Both the *technique* of resolution of the proposed situation and the *technology* that justifies it are implicit in it. Now, let us consider the existence of fractional numbers (quotients of integers) and their properties, which Chevallard consider to be another technology that justifies another technique different from the one shown in the statement:

¨This is how fractional number technology (integer quotients) allows us to generate a technique that classifies what was previously seen in relation to caramel prices and which specifies the following discursive scheme: "If *a* things cost *b* francs, then *x* things, that is x / a times a things, will be worth x / a times more, that is, x / a times b francs. "Thus we can say: "11 caramels cost 11/8 times more (than 8 caramels), that is 11/8 times 10 francs (= 13,75 francs)"; "and by a daring extension of the meaning of the expression:" 3 caramels cost 3/8 times more (than 8 caramels), that is 3/8 times 10 francs (= 3.75 francs). (It will be indicated that it is: 3/8 x 10 francs = 11 / 8x10 francs - 8 / 8x10 francs = 13.75 francs - 10 francs = 3.75 francs). More correctly, it will simply be said that "x things, is x / a times a things", etc¨. (Chevallard, 1998, p. 4).

When we reflect on Chevallard's passage in a Husserlian approach, we see that the constitution of a new "view" happens, which embraces what is already known and that stands as a finding: "If *a* things are worth *b* francs, then *x* things, i.e., $\frac{x}{a}$ times a things, will be worth $\frac{x}{a}$ times more, i.e., $\frac{x}{a}$ times b francs, which is described in Husserl as *transition syntheses* constructed and under construction, when articulated to other *transitional synthesis* of knowledge. As in Chevallard, *technologies* are fruit and triggers of the development of theories, which favour new techniques and generalised findings, in an ongoing process of development. And so,

¨Since the notion of *function*- and more particularly the notion of *linear* function - is available, as well as of the usual functional notations, we can return to the problem of the 3 caramels in these terms: being *f* linear, if *f* (8) = 10, then *f* (3) = *f* (3/8 × 8) = 3/8 × *f* (8) = 3/8 × 10 =... ¨. (CHEVALLARD, 1999, p. 227)

Thus we see the calls, by Husserl, *of transitional syntheses*, bearers of the historical *Apriori*, consistent with the mathematics body of knowledge, instituted under the epistemological and pragmatic lens proposed by

Chevallard, who describes them from the function that technique, technology and theory exert on the body of mathematical knowledge.

From our standpoint, to redeem the epistemology, even if it is only in its technological aspects, as proposed by Chevallard in the anthropological theory of the didactic in the teaching processes, is of vital importance for us to take another step back, as prompted by Husserl in referring *to the original evidence* and its relationship with the axioms in order to rescue the sense of world that mathematical statements translate, and that these senses of the world can be the foundations of a dialogue between mathematics and the assertions from the other exact sciences and humanities, constituting a ground for the discussions that are already taking place in the so-called *noosphere* by Chevallard and with that, perhaps, resignify the significance traditionally put in the word contextualisation, launching new perspectives/views into the didactic transposition of mathematical knowledge.

What has hitherto been exposed about the possible approximations and estrangements of both theories studied is addressed, in terms of the Hursserlian theory, to the noematic, i.e., to what is being placed as object of the relationship - the established mathematics or the *wise knowledge* - both in the theory of the didactic transposition as in the anthropological theory of the didactic (ATD), which culminates in the comprehension of the didactic as all artefacts, works, i.e., institutional constructions that base and compose didactic choices (Chevallard, 1996) expressed in a discourse, bearer of a *logos.*

But it is when the author affirms that: "the ATD places the mathematical activity and, therefore, the activity of studying mathematics, in the collective of human activities and social institutions" (Chevallard, 1998, p. 1) and that human activity can be considered in a single model he calls praxeology, is that we see to emerge a possible link between the two theories in terms of what Husserl called noesis in the noesis- noema relationship, which deals with the intentional act of consciousness in the disposition of the subject to see something, that is, to understand something.

We will start explaining this statement based on the answers from the solo texts related to the question: In what aspects do the authors understand human activity?

In the theorisation of human activity - described by Chevallard as a praxeology in the perspective of the accomplishment of tasks - are the mathematical activities and the activities related to their didactic conduction. So, it is composed of two parts: one, that focuses on the mathematical organisation of tasks aimed at teaching practice to be made and analysed under the focus of the articulations between the proposed resolution technique, the technology that justifies the technique and the theory that founds it; the other part deals with the didactic organization of its implementation consisting of didactic moments described as moments of studies: 1) to submit the object to be studied evidencing its *raison d'être*, i.e., expose the motives which led to its construction; 2) to explore the type of task and its elaboration technique; 3) to weave relationships with other technical and theoretical moments already studied previously; 4) to improve

the technique used in the resolution of the task; 5) to institutionalise the math organization, i.e., distinguish the vital elements from those that can be discarded; and 6) to assess the value of mathematical organization institutionalised. Therefore, what is being evaluated here is praxeology itself, because, "behind the classic evaluation of personal relationships, that is, behind the evaluation of "people", "is the evaluation *of the norm itself* - the institutional relationship that serves as a reference¨. (Chevallard, 1998, p. 22-23).

In this way of composing praxeology, human activity is thought in terms of the execution of tasks, without sticking to the processes that enable to grasp the knowledge implied in the tasks, i.e., it does not, in itself, include the "how" the study of technology as a rational discourse on a technique happens, understood as *logos* and as something that varies throughout the historicity of the mathematical objects or, still, does not stick to processes that could possibly subsidise the search for the most appropriate mathematical technologies and theories to the requirements imposed by the educational system. This theorisation shows a model which, in our opinion, in being followed and put into practice, will merge inevitably with other theories that focus on the above-mentioned processes, so that they can be put into practice aiming at reviewing and analysing the didactic constructs.

Husserl (2012) in *Investigações lógicas/Logical Investigations*, on the other hand, turns his attention to human activity, when confronting the rational mathematical discourse as *logos* from the point of view of the noesis-noema relationship as acts of conscience that describe the relationship of this encounter in different layers, which go from the act of signifying what was said in the discourse, such as the acts of filling of sense through intuition, until acts of judgment that utter the assertions put in the mathematical discourse as objectifying acts, intertwined with logic. And the latter understood as the science of meaning. Since:

¨[...] in the sphere of arithmetic-symbolic thought and calculation, one does not operate with signs without meaning. It is not the "simple" signs, in the sense of the physical, the signs detached from all and any meaning, which subrogate the animated original signs of arithmetic meanings; or rather, what subrogates the arithmetic significant signs are the same signs, but taken in a certain operative meaning or game¨. (Husserl, 2012, p. 57)

In this work, Husserl, in investigating the meanings of scientific propositions, culminates in the critical weaving of the misunderstandings of the terms: *representation* and *content*, when seeking meaning in the expression "what representation represents", which was deepened in the *Sexta Investigação Lógica/Sixth Logical Investigation* (2000). A theme that deserves, in the scope of phenomenology, a bigger explanation space than this article allows us. In Husserl's words (2000):

¨A **meaningful representation** has no essence in itself. However, we attribute to it, in an **improper sense**, a certain essence, when it admits being filled completely by one of the multiple possible intuitions of that essence; or, what makes no difference, when it has a "filling sense"¨. (Husserl, 2000, p. 101)

Later, Husserl (2012; 1936) seeks to understand the historicity of ideality (for example, the historicity of mathematical objects) in its flow of maturation in a unit of time called the *living present*, which is not cloistered in particular *agoras*. It describes the construction and maturation of the mathematical object as an ideality in formation that encompasses subjective, intersubjective and objective aspects:

¨Every singular perceived has in the continuous perception, which extends in the far or near flow, sooner or later interrupted, an own and extensive movement of giving oneself, and concomitantly a horizon of joint opinions of characteristics such as that for which the real shows itself or becomes shown, when it would already be given as a complement to the anticipation of the horizon. This is empty, relative, indeterminate, and only exceptionally in an expected start as the anticipated future of giving oneself, which is yet to be completed. To which every singular view of reality has a joint opinion as horizon¨. (Husserl, 1936, p. 251)

In this way, objectivity, from the point of view of phenomenology, is reached in a flow of maturation that transmits the a *priori*, given in the perception of the world in the noesis-noema relationship, as intuition of essence - as called in *Investigações Lógicas/Logical Investigations* and later in *Crise da Ciência Europeia/Crisis of European Science*, called *original evidence*, transmitted as a *universal historical Apriori* when adequately complemented without losing the sense of the life-world given in the *original evidence* and agreed upon based on the *intersubjectivity* that makes up the nuclear, the transitional syntheses, as formal ontologies, those which are expressed via language, in the case of mathematics, via mathematical language.

We have made here a succinct account of questions addressed by Husserl on the formation of the mathematical ideality. From him, we can affirm that the teaching of mathematics and the didactic questions were not part of his preoccupations, even because he proposes a phenomenological philosophy and seeks to meet the precepts of this region of knowledge. But we understand that philosophy, which emerges from the Husserlian phenomenological reduction, brings fundamentals and methodology so that we can think about the encounter of man with mathematics established in situations of teaching and learning mediated by didactic choices.

On the other hand, Chevallard did not intend to address any of Husserl's very carefully considered aspects, even acknowledging how important some of them were, such as the study of questions involving representations and "how" the construction of the mathematical knowledge is from the point of view of the relationship of the subject with the world and from the point of view of its constitution and the establishment of the scientific mathematical knowledge, that he called *wise knowledge*.

In this perspective, we see the possibility of meaning-filling approximations between the two theories, since both authors assume the body of mathematical knowledge in its technical-logical aspects and that the didactic moment of the encounter between the student, the teacher, and the object of the mathematical knowledge, as human construction, is assumed in terms of a relationship.

We will explain this in the next item.

Resuming the view of the theoretical links

Based on the possibility of the confluence between the empirical sciences and the eidetic sciences that can be woven by ideations, we understand the possibility of the emergence of a phenomenological pedagogies of mathematics as a science of essences that retakes the model of Chevallard's theorization, that is the relationship of the subject with the object, taking as object the established mathematics, but looking for the essences of this relationship, that is, exposing "how" it occurs, having as basis the Husserlian studies on the acts of conscience implicit in the encounter of the man with the mathematical discourse, and how these acts compose the intentional experience that seeks the understanding of this discourse in its peculiarities. What justifies the development of a didactic of its own, different from others that only concern man, referring to the vast labyrinth of the human constitution or, in other extreme, those that totally disregard human elements, focusing only on mathematical knowledge and in its internal organisation. We thus envisage a didactic of mathematics that can account for the encounter of man with mathematics in a teaching and learning situation, where knowledge can be constituted.

With this theoretical background that encompasses not only what Chevallard calls epistemology of mathematics, but also everything that can drive our analysis by the trail of a view of logic that is attentive to ontic issues of the world knowledge interwoven with the human, we understand that, starting from the math organisation proposed by Chevallard in his praxeology, subdivided in technique, technology and theory, subdivisions that, in Husserl's view, make up a *synthesis of transition*, implied in a mathematical task to and from the classroom, we can perform a *phenomenological reduction* to restore the sense of world implicit in the knowledge to be taught when questioning it retrospectively with the objective announced by Husserl (1997), because

¨Only the revelation of the general essential structure found, wholly as such in our - and then in that of any - past or future historical present, in which we live, in which our whole humanity lives, in relation to its total essential structure, only this revelation can actually enable a understandable, intelligent, in a sense scientific, history. This is the concrete historical *Apriori*, which takes all that exists as tradition and transmission, in a historical being of the past or historical Being of the future or in its essential being¨. (p. 457)

We understand that, in a way, Chevallard also seeks, in the history of mathematics, the techniques, technologies, but we think it is necessary to undertake a methodological movement, led by the *phenomenological reduction*, that can bring up the historicity of these elements, i.e., the reasons for the articulations and bindings that make them emerge, so that, when we talk didactically about the reasons of being of the mathematical object of the task to be performed, we are not restricted to why it is there, but that this is extended to the elucidation of its essential aspects, that will not only be the basis for the teaching of mathematics, but that can meet

the demands of the educational system and educational policies that design the pedagogical path to humanising the human.

When the "how" is clarified for the relationship of the subject with the established mathematics, which, for the phenomenology, has the duration of a lived experience, covering not only the initial moment of a contact, but the internal exploration carried out by the human of the apprehended in the initial moment, we think that the conversations with the other areas of knowledge in determining the didactic operated in the noosphere, described by Chevallard, could be facilitated, since the arguments about what to include from the mathematical content and how to create and implement the didactic tasks will be oiled by the characteristics and essences of the human and sense of the world.

All of what was said, even if with solid foundations stemming from the work of the two authors studied, are views of a creation called here didactic of phenomenological mathematics to be constructed.

Everything that has been said, even though with solid bases arising from the work of the two authors studied, constitutes a glimpse of the creation called here Phenomenological pedagogies of Mathematics to be constituted, that contemplates not only the epistemology of the established mathematics, but also its constitution linked to a human making that not only speaks of science, but also of the human itself, of the meaning of the world and how this meaning is complemented in the construction of mathematical objects and in human decisions that occur in the noosphere. In this way, the phenomenological pedagogies of mathematics will be guided not only by the epistemology of mathematics and ways of teaching derived from its episteme, but also by the meaning of the world, learned and elaborated by the human being, who becomes the founder of the construction and constitution of human knowledge and the acts of human nature that constitute them.

Conclusions

Human life and the duration of its products are so broad that a single science cannot embrace. Being conscious of the competencies of the theories seems to us a healthy attitude to every researcher, even if we cannot reach all of them.

In this research, we pursued the approximations and estrangements between Chevallard’s and Husserl’s foundations and, as described, from the analysis, some regions of inquiry that remain untouched by the authors will emerge. Hence, it becomes important that once we have foundations - if not common and identical, but that dialogue with each other and that allow for convergences, giving rise to coherent complements - we dwell on them so that we can open possibilities not yet seen. Such is the f the phenomenological pedagogies of mathematics.

Why phenomenological pedagogies of mathematics? Why not didactics of mathematics in a phenomenological approach?

Within the research area of mathematics education, the didactics of mathematics based on Chevallard's works and developed in the French school is recognised as an experimental science and, as such, can be

developed assuming other theories as a foundation, adopting its methods, as long as these are compatible with their way of doing science and thus creating their own approaches, in the sense of reinvigorating the epistemological aspects of mathematics in a teaching and learning situation.

However, when we suggest the creation of a phenomenological pedagogies of mathematics, we are referring to the creation of an eidetic science that seeks the essence of man's encounter with mathematics in a teaching and learning situation, in the sense of invigorating the sense of world that is implicitly consistent with mathematical knowledge. For that, its method of analysis will be that of *phenomenological reduction*, therefore it will have to produce its results to, only afterwards, confront them with the results of other didactics of mathematics, such as those in the empirical sciences.

We may ask ourselves: If so, why seek inspiration in Chevallard's work?

The answer to this question seems to be now: Because Chevallard's work indicates a way to be followed in the recovery of the episteme of the body of the mathematical knowledge in the tasks that are offered to the apprentice, in the didactic doing of the mathematics classes. From there, we can go on retroactively in search of the *sense of world* put in the object of knowledge and by the acts of consciousness that guide us to their understanding. Further reflections phenomenologically constructed in the field of mathematics education may contribute to better understand the issues that are part of the interdependencies of the didactics of mathematics with education and its subareas, as well as other areas of knowledge, such as anthropology itself.

We thus see Chevallard's and Husserl's thinking being architected. While Husserl "shakes" the box of knowledge construction in a philosophical perspective, reflecting on the knowledge intertwined with the human acts that extrapolate the individual existence, weaving a view of man, world, reality, representation, Chevallard offers his view on how the established knowledge has been idealised as knowledge to be transmitted in the layers of teaching as didactic, trying not to lose the mathematics epistemological roots which, according to our understanding, maintain the bonds of mathematics with logic.

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